YASHWANTRAO CHAVAN MAHARASHTRA OPEN UNIVERSITY, NASHIK

(Estd. By Government of Maharashtra)
Dnyangangotri, Near Gangapur Dam, Govardhan, Nashik-422 222
Home Assignment

ज्ञानगंगा घरोघरी

Academic Year - 2024 - 2025

Name of the Programme & Prog. Code	V151 - M.Sc. in Mathematics(with Credits) - Open and Distance Learning - 2023 Pattern - NEP - Year-2 Semester III & V151
Year / Semester	Year-2 / Semester III
Name of the Course & Course Code	Discrete Mathematics (MAT606)
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Name of the Study Centre & Code	JIJAMATA GRAMIN VIKAS SHIKSHAN SANSTHA ,JIJAU ARTS, COMMERCE AND SCIENCE COLLEGE. WARUD & 2370A
Date of Submission	20/12/2024 — No of Pages: 68

General Instructions

1	Home assignment is useful for self-analysis of your preparation of the final examination and progress
2	Read all the questions and their marks in the home assignment carefully understand the Definitions and concepts properly
3	Read carefully the text, syllabus and summary related to home assignment.
4	Do not copy the points, matter from the text while writing home assignment mentions your one point /opinions whenever necessary
5	Your answer should include how you would apply the knowledge in real life situations
6	Use A4 size lined papers for writing home assignment by taking printout from this file only
7	Write each new answer on a different page.
8	Printed or typed answers are not allowed
9	The marks of Home assignment and corrective instructions will be sent after checking home assignment
10	Present the Home Assignment by following the instructions and rules. Read carefully all the instructions and rules before any correspondence or communication with the university
11	**For each theory course of 4 credits its carry 30 marks, there are Two Home Assignments:** i) **Home Assignment 01 (HA01) is of Twenty marks** consisting of 2 Short Answer Questions carrying 5 marks each and 1 Long Answer Question of 10 marks. ii) **Home Assignment 02 (HA02) is of Ten marks** consisting of total 10 Objective Type Questions (MCQ) carrying 1 mark each.
12	**For each theory course of 2 credits, there is Only One Home Assignment of Fifteen Marks** consisting of 1 Short Answer Question of 5 marks and 1 Long Answer Question of 10 marks.
13	Student have to write the Home assignment 02 (MCQ) on separate page. Home Assignment 02 also to be uploaded along with Home assignment 01.

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Q.NO.

1 Solve following long answer question

a) Define lattice, furthermore, let P be a poset such that inf [a,b] and sup[a,b] exist for any a,b in P. letting $a \wedge b$ = inf [a,b] and $a \vee b$ sup [a,b] Then prove that $(P, \wedge, \vee)$ is a lattice. Also show that the, partial order on P.

Ans- Let L be a non-empty set closed order under to binary operations called meet, and join, wich are denoted by $\wedge$ and $\vee$ resp.

proof - Let

P be the poset such that inf [a,b] and sup [a,b] exist for any a,b in L.

Denote $a \wedge b$ = inf [a,b] and $a \vee b$ = sup[a,b]
To show
$(P, \wedge, \vee)$ is a lattice as.

(P) Commutative law:-

for any $a, b \in P$ we have

$a \wedge b$ = inf (a, b)

= inf [b, a]

= $b \wedge a$

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Similarly, we can show $a \vee b = b \vee a$

② Associative law:-

First we prove $a \vee (b \vee c) = (a \vee b) \vee c$
i.e. $g = \sup \{a, \sup\{b, c\}\}$ and
$h = \sup \{\sup\{a, b\}. c\}$

we obtain
$\therefore a \leq g$ and $b \vee c \leq g$

furthermore $b \vee c \leq g$ means that
$b \leq g$, $c \leq g$

$\therefore$ we have $a \leq g$ and $b \leq g$ and $a \vee b = \sup\{a, b\}$

we obtain
$a \vee b \leq g$ wich together wich $c \leq g$
leads to $(a \vee b) \vee c \leq g$

But $h = (a \vee b) \vee c$

$\therefore h \leq g$

In a similar manner, we can show that
$g \leq h$

$\therefore$ But antisymmetry property of a partial ordering relation, we conclude that.

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$g = h$

and hence, $a \vee (b \vee c) = (a \vee b) \vee c$

Now by the duality principal. we have $a \wedge (b \wedge c) = (a \wedge b) \wedge c$

③ Absorption law:-

for any a, b in P

Concider $a \wedge (a \vee b)$

$= \inf\{a, b \vee b\}$

$= \inf\{a, \sup\{a, b\}\}$

$= a$ similarly, we can show that

since $a \leq \sup\{a, b\}$

$a \vee (a \wedge b) = a$

Thus $(P, \wedge, \vee)$ is a littice.

Define a relation '$\leq$' on P produced by a lattice $(P, \wedge, \vee)$ as, $a \leq b$ if and only if

$a \wedge b = a$

OR

$a \vee b = b$ then in the previous theorm we have proved that this relation '$\leq$' is a partial order lattice $(P, \wedge, \vee)$.

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2 Solve the following ~~sobor~~ short ans question

i) there are no restrictions.

ii) 2 books are always together

iii) 2 books are never together

Ans Soln:- Let

i) No restrictions:

When there are no restrictons, we simply need to arange all 5 diffrent books on the shelf. The number of ways to arrange 5 books is given by the factrial of 5.

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

So, the number of ways to arange 5 books no restrictions is 120,

ii) 2 books are always together

if 2 books are always together, we can treat those 2 books as a single 'block', Now, insted of 5 books, we have 4 units to arange: the "block" of 2 books and remaining 3 individual books.

① The number of ways to arange these 4 units is 4!

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(2) Within the "block" the two books can be arranged in 2! ways.

Thus, the total number of arrangements is

$4! \times 2! = 24 \times 2 = 48$

So, the number of ways to arrange the books when 2 books are always together is 48

iii) 2 Books are never together:-

If two specific books are never together, we can first calculate the total number of unrestricted arrangements and then substract the number of arrangements where the two books are together.

(1) The total number of unrestricted arrangments from (1) is $5! = 120$

(2) The number of arrangment when the two books are together from (2) is 48

Therefore
The number of arrangments where the two books are never together is

$5! - 4! \times 2! = 120 - 48 = 72$

So
The number of ways to arrange the books where 2 books are never together is 72

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2

b) Prove the number of vertices of odd degree in a graph is always even.

Ans Proof :- Let.
If we consider the vertices with odd and even degress separately, the gauntity in the left side of eqⁿ, can be expressed as the sum of two sums. each taken over verties of even and odd degress.

Separately, as given below

$$\sum_{i=1}^{n} d(v_i) = \sum_{\text{even}} d(v_j) + \sum d(v_k) \quad \text{--- (1)}$$

Since, the left hand side in eqⁿ (2) is even, and the first expression on the right hand side is even (as it is a sum of even numbers), the second expression must also be even

$$\sum_{\text{odd}} d(v_k) - \text{an even number} \quad \text{--- (2)}$$

Because in eqn (2) each $d(v_k)$ is odd, and the right hand side is even, we want the total number of terms in the in the sume of left hand side must even to make the sum and even number.

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Therefore,
The number of vertices of odd degree in a graph is always even, Hence proved.

Home Assignment - 2

Solve following multiple choice question

1) There are — different arrangments can be made by using all the letters of the word 'Thursday'.

Ans:- b) 8!

2) In a Boolean algebra an elements —
Ans:- c) has unique complement

3) The number of ways of selecting a football team of players from 15 players wich include two particular players are —

Ans:- b) $^{14}C_{10}$

4) In a jar there are 5 black marbles and 3 marbels. two marbels are picked randomly one ~~other~~ after the other without replacment. what is the propibability that both the marbels are black.

Ans:- b) 5/14

5) If for some positive intiger k, d(v)=k for every vertex v of graphs G, then G is —
Ans:- b) k- regular

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6) A graph containing n vertices is a tree if and only if its chromatic polinomial

Ans:- a) $P_A(\lambda) = \lambda(\lambda-1)^{n-1}$

7) If T is a tree with 99 vertices then it has precisely ______

Ans:- b) 98 edges

8) The number of vertices n, in a binary tree is always ______

Ans:- d) n

9) A vertex v of a tree T is cut vertex if and only if ______

Ans:- d) d (v) >1

10) A complete graph of n-vertices is

Ans:- c) n- chromatic

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